Aim:

- To study the transient response in storing an electrical charge on a capacitor in an RC circuit.
- Also to study the transient decay of an initial charge on a capacitor through a resistor.
- To understand the time constant in an RC circuit and how it can be changed.

Background & Theory:

A capacitor has the ability to store an electrical charge and energy. The voltage across the capacitor is related to the charge by the equation \( V = \frac{Q}{C} \) for steady state values, or expressed as an instantaneous value,

\( \frac{dv}{dt} = \frac{dq}{C} \)

By definition \( i = \frac{dq}{dt} \) or \( dq = idt \). Therefore

\[
\frac{dv}{dt} = \frac{1}{C}idt \quad \text{or} \quad v = \frac{1}{C}\int idt
\]

The derivation of the transient responses of both the capacitor current and voltage in an RC circuit when a source voltage is suddenly applied to that circuit is shown below. Note that the time constant \( \tau = RC \).

The step response of an RC circuit can be analyzed using the following circuit:
Immediately after the switch closes, KVL requires that -

If we differentiate (1) with respect to \( t \), we get

\[
R \frac{di}{dt} + \frac{i}{C} = 0
\]

The other two terms drop out because they are constants. Now divide thru by \( R \) -

\[
\frac{di}{dt} + \frac{i}{RC} = 0 \quad \Rightarrow \quad \frac{di}{dt} = -\frac{i}{RC} \quad \Rightarrow \quad \int_{i_0}^{i(t)} \frac{di}{i} = -\frac{1}{RC} \int_{0}^{t} dt
\]

The voltage across the capacitor at \( t = 0 \) (\( V_c \)) will be zero because there cannot be an instantaneous change in voltage across the capacitor. Therefore, the initial current in the circuit will be as follows:

\[
i(t) = I_0 e^{-\frac{t}{RC}}
\]

\[
i(0) = I_0 = \frac{V_s}{R}
\]
Substituting (8) into (7) and plotting Normalized current = $i(t)/I_o$, versus Normalized time = $t/RC$.

**RC charging current**

![Graph showing normalized current versus normalized time](image)

Note that the time constant ($t = \tau = RC$) occurs at 36.8% of $I_o$ or 0.368 $V_s/R$.

We also know that

$$i_c(t) = C \frac{dV_c}{dt} \quad \Rightarrow \quad V_c(t) = \frac{1}{C} \int_0^t i_c(x) dx + V_c(0)$$

Substituting $i$ from (7) and $I_o$ from (8) into (9),

Putting in limits and simplifying gives:

$$V_c(t) = \frac{V_s}{RC} \left[ -RC e^{-\frac{t}{RC}} + RC \right] = V_S - V_se^{-\frac{t}{RC}}$$

$$V_c(t) = V_s \left( 1 - e^{-\frac{t}{RC}} \right)$$  Capacitor voltage at any time $t$ when charging from zero

$$V_c(t) = V_se^{\frac{-t}{RC}}$$  Capacitor voltage at any time $t$ when discharging to zero
Plotting normalized voltage ($V_c/V_S$) versus normalized time ($t/RC$)

![RC Charging Voltage](image)

Note that the time constant ($t = \tau = RC$) occurs at 0.632 $V_S$

**Apparatus:**

- $V_{dc} = 12V$
- Breadboard
- Digital Multimeter (DMM)
- Fixed Resistors: 68kΩ, 100kΩ
- Electrolytic Capacitor 470 µF
- PSpice software

**Method:**

*Transient Response of RC circuit when capacitor is single*

1. Calculate time constant $\tau = RC$ for a series RC circuit having $C = 470 \mu F$ for $R = 68k\Omega$ and $R = 100k\Omega$ to gain a perspective of how long the transients will take. A capacitor will be mostly charged or discharged after five time constants, $5\tau$. This is also called as transient period. Record in Table 4-1.

2. Construct the circuit of Figure 4-2 using the values of $R$ and $C$ given in step1. You will use a jumper wire for the switch to connect the resistor either to the voltage source or to the reference node (ground). Be sure to connect the negative side of the electrolytic capacitor to ground.

![Figure 4-2](image)
3. Set $V_s$ at 12 volts. Leave the jumper wire in the discharge position until the voltage across the capacitor stabilizes at 0 volts. (Note that if the capacitor has been charged before, a wire temporarily shorting out the capacitor will speed up this process. Do not use your bare hands).

![Figure 4-3](image)

4. Then put the jumper wire in the charge position.
5. Record $V_c$ every 20 seconds up to 4 minutes (240s). Then leave the switch in up position until the voltage $V_c$ stabilizes at the maximum value (when the second digit of the multimeter is no longer changing over 30 second period) and record that value in Table 4-2.
6. Note: One person will need to call off time and the other person read the meter and write down voltage. You may need to practice and repeat the steps until you establish a good procedure for taking data.
7. Next put the jumper wire in the discharge position and record the capacitor voltage $V_c$ at the same time intervals as in step 5.
8. Repeat items 3 to 6 with $R = 100k\Omega$. Record the values in Table 4-2.

**Transient Response of RC circuit when capacitors are in parallel**

1. Construct RC circuit of using one $R = 100k\Omega$ and two $C = 470 \mu F$. Now, the capacitor are in parallel.
2. Find the total capacitance. For parallel capacitors, the total capacitance is $C_T = C_1 + C_2$. Calculate the transient period $5\tau$. The charging and discharging of the capacitor will stabilize at this period.
3. Repeat step 3 and 4 in the first experiment.
4. Repeat step 5 and record that value in Table 4-3.
5. Repeat step 6 and 7 and record that value in Table 4-3.

**Transient Response of RC circuit when capacitors are in series**

6. Construct RC circuit of using one $R = 100k\Omega$ and two $C = 470 \mu F$. Now, the capacitor are in series.
7. Find the total capacitance. For series capacitors, the total capacitance is $\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2}$.

Calculate the transient period $5\tau$. The charging and discharging of the capacitor will stabilize at this period.
8. Repeat step 3 and 4 in the first experiment.
9. Repeat step 5 and record that value in Table 4-4.
10. Repeat step 6 and 7 and record that value in Table 4-4.
Report:
Information for each formatted categories must be provided as outlined in the introduction.

i. Tabulated results – 2 pts.
   - Measured value – check for trend validity, difference between two resistors
   - Theoretical table using the formulas for charging and discharging

ii. PSpice Simulation – 5 pts.
   - Schematics and corresponding graphs for resistor and capacitor in transient analysis
   - Schematics and corresponding graphs for double energy circuits (overdamped RLC circuit, critically damped RLC circuit and underdamped RLC circuit.

iii. Discussion and Conclusion – 3 pts
   - 2 manually extrapolated graphs indicating experimental \( \tau \)
     i. Charging curves
     ii. Discharging curves
   - Theoretical elaboration based on the handouts given
# Table of Results

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<th>Time constant</th>
<th>R=68KΩ</th>
<th>C=</th>
<th>τ</th>
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<td>R=100KΩ</td>
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Table 4-1

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<th>Vc R=100KΩ</th>
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Table 4-2
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<th>Time interval (s)</th>
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Table 4-3
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Table 4-4